Stochastic Approach to Estimate Interlaminar Losses of Electrical Sheets

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Random galvanic contacts are formed between the edges of the electrical sheets when punched and pressed. In this paper, finite element formulation is introduced with a suitable boundary condition to account for these interlaminar contacts. The spatial variation of the conductivity at the edges of these electrical sheets is discretised using the Karhunen-Loeve expansion and propagated through the finite element formulation. The response to be obtained is approximated using polynomial chaos expansion. Then, the additional losses due to the interlaminar contacts are estimated from the solution obtained from different stochastic methods. The accuracy and computation time of these novel stochastic approaches are also discussed. The accuracy and computation time of these novel stochastic approaches are also discussed.

***Index Terms*—Finite element analysis, Monte Carlo method, polynomial chaos expansion, random field, stochastic process.**

I. Introduction

PUNCHING and pressing of electrical sheets forms burrs and deteriorate the magnetic properties of their edges. These burrs deteriorate the insulation of adjacent sheets and make random galvanic contact between the sheets [1]. These galvanic contacts are stochastic in nature. There are different stochastic models available in the literature to account for the uncertainties. All these studies are mostly related to the field of mechanical and civil engineering. Uncertainties in the model geometry and different stochastic methods are discussed in [2]- [4]. In the field of electromagnetism, stochastic studies are ongoing and there are some studies done to quantify the uncertainties introduced in the magnetic properties of electrical sheets due to manufacturing effects [5]. This stochastic behaviour of magnetic properties is used as input in stochastic finite element models [6] to study the variability in the losses of electrical machine.

In this paper, uncertainty introduced due to the formation of burrs during manufacturing process is modelled by using electrical conductivity at the edges of the sheets as a random field. Firstly, a deterministic finite element formulation is introduced to model the conducting edge of UI electrical sheet as shown in Fig. 1. Then, the random field is dicretised and implemented in the finite element method. Finally, the losses due to the conducting edge in the UI sheet are estimated and the computation time and accuracy of different stochastic methods are studied.

II. Methods

A. Model formulation

The conducting edge in a two-dimensional finite element method is modelled using the conventional $\boldsymbol{A}-\phi$ formulation with an additional integro-differential equation. The field equation solved in UI sheet is given by,

$$-\nabla\cdot\left(\frac{1}{\mu}\nabla\boldsymbol{A}\right)+\sigma\frac{\partial\boldsymbol{A}}{\partial t}-\sigma\frac{u}{l}\,\mathbf{e}_{\mathrm{z}}=\boldsymbol{J}_{\mathrm{s}}\,. \tag{1}$$

Equation (1) is valid for all the conducting ($\sigma\neq 0$), non-conducting ($\sigma=0$) and source region. $\boldsymbol{J}_{\mathrm{s}}$ is source current density, μ is the permeability of iron and σ is the electrical conductivity of edges of electrical sheets. u is the induced voltage at the conducting edges due to time varying magnetic field, $\mathbf{e}_{\mathrm{z}}$ is the unit vector parallel to z direction. The presence of galvanic contacts along the edges causes the induced current at one edge to return it from the opposite edge. Hence, the net induced current flowing through the burred edges is forced to be zero.

The induced current is given in (2), where surface (S) is the area of burred region in which induced current density is perpendicular. Then, the net current is given by,

$$I=\int_{\mathrm{S}}\sigma\left[-\frac{\partial\boldsymbol{A}}{\partial t}+\frac{u}{l}\,\mathbf{e}_{\mathrm{z}}\right]\cdot d\mathrm{S}=0, \tag{2}$$

$$\sigma\frac{u}{l}\mathrm{S}=\int_{\mathrm{S}}\sigma\frac{\partial\boldsymbol{A}}{\partial t}\cdot d\mathrm{S}. \tag{3}$$

Finally (1), is written as,

$$-\nabla\cdot(\nu\nabla\boldsymbol{A})+\sigma\frac{\partial\boldsymbol{A}}{\partial t}-\frac{1}{\mathrm{S}}\left(\int_{\mathrm{S}}\sigma\frac{\partial\boldsymbol{A}}{\partial t}\cdot d\mathrm{S}\right)\mathbf{e}_{z}=\boldsymbol{J}_{\mathrm{s}}\,\mathbf{k}\,. \tag{4}$$

The additional integro-differential equation that ensures the net induced current to be zero forms a denser matrix and increases the computation time. Equation (4) is the deterministic mathematical equation where uncertainty in the conductivity is propagated. Hence, computationally efficient and accurate stochastic method is required.

B. Stochastic finite element formulation

1) Series Expansion

The conductivity $\hat{\sigma}(\mathbf{x},\theta)$ at the edges is considered as a random field, θ is used as the symbol to represent it as a random quantity. It is then approximated as a spectral

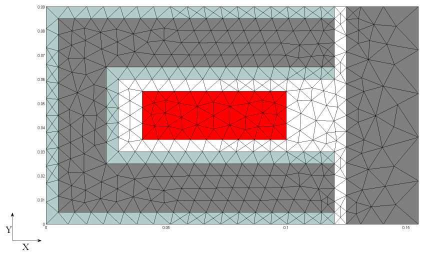

Fig. 1. UI electrical sheet with conducting edge

decomposition of its auto covariance function $\rho(\mathbf{x},\mathbf{x}')$ which is truncated after the M^{th} term and is given by,

$$\begin{aligned} &\hat{\sigma}(\mathbf{x},\theta) = \sigma_\mu + \sum_{k=1}^{\mathrm{M}} \sqrt{\lambda_k}\phi_k(\mathbf{x})\zeta_k(\theta), \\ &\text{where,} \\ &\lambda_k\phi_k(\mathbf{x}) = \int_{\Omega_{\mathrm{RF}}} \rho(\mathbf{x},\mathbf{x}')\phi(\mathbf{x}')d\Omega_{\mathbf{x}'}. \end{aligned} \tag{5}$$

This Karhunen-Leove (KL) expansion basically requires the solution of a eigenvalue problem. $\lambda_k, \phi_k(\mathbf{x})$ are the eigenvalues and the eigen function, respectively. $\mathbf{x}$ is the two-dimensional domain of geometry, σ_μ is the mean value and ζ_k are the basis of independent random variable. The solution of eigenvalue problem is obtained considering the exponential autocorrelation function [7].

2) Spectral stochastic finite method element (SSFEM)

The conductivity is discretised and then propagated through the deterministic mathematical formulation. In our problem, the magnetic vector potential is the response and it is approximated using a polynomial chaos expansion (6). The coefficients ($\boldsymbol{A}_j$) are to be determined and Ψ_j are the basis of hermite polynomials.

$$\begin{aligned} A &= \sum_{j=0}^{\text{P-1}} A_j\ \Psi_j(\theta), \\ \mathrm{P} &= \sum_{q=0}^{\mathrm{o}} \frac{(\text{M-1}+q)!}{q!(\text{M-1})!}. \end{aligned} \tag{6}$$

The expansion size of the polynomial is given by P and o is the order of the polynomial.

Finally, after discretising (4) in space and implementing the expansion of conductivity, the system of matrices (7) is obtained. For simplicity, symbol θ is omitted and only time harmonic case is considered.

$$\left[\boldsymbol{S}_{ij} + \mathrm{j}\omega\left(\boldsymbol{T}_{ij} + \boldsymbol{I}_{ij}\right)\right]_{\mathrm{n}\times\mathrm{P},\mathrm{n}\times\mathrm{P}} \cdot \left[\boldsymbol{A}_j\right]_{\mathrm{n}\times\mathrm{P},1} = \left[\boldsymbol{J}_j\right]_{\mathrm{n}\times\mathrm{P},1}, \tag{7}$$

where, i,j = 0,..., P-1. The total dimension of the matrix is $\mathrm{n}\times\mathrm{P},\mathrm{n}\times\mathrm{P}$ and n is the total number of degree of freedom in the finite element method. The discretised conductivity is assembled as given by,

$$\begin{aligned} \boldsymbol{T}_{ij} &= \bar{\boldsymbol{T}}\Psi_i\Psi_j + \sum_{k=1}^{\mathrm{M}} \zeta_k\Psi_i\Psi_j\,\boldsymbol{T}_k, \\ \bar{\boldsymbol{T}} &= \int_{\Omega_{\mathrm{e}}} \sigma_\mu\,\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}\,d\Omega_{\mathrm{e}}, \\ \boldsymbol{T}_k &= \int_{\Omega_{\mathrm{e}}} \sqrt{\lambda_k^{\mathrm{e}}}\phi_k^{\mathrm{e}}\,\boldsymbol{N}^{\mathrm{T}}\,N d\Omega_{\mathrm{e}}. \end{aligned} \tag{8}$$

Similarly, $\boldsymbol{I}_{ij}$ and $\boldsymbol{S}_{ij}$ are assembled. However, reluctivity which is not a random quantity needs to be expanded to M terms for the sake of computational simplicity.

C. Monte Carlo Method (MCM)

In Monte Carlo method, finite element formulation (4) is solved for all possible samples of conductivity. The conductivity at the edges of the sheets are taken randomly from thousands of sample spaces. The samples are generated from MATLAB. The interlaminar loss is calculated using both MCM and SSFEM.

III. Results and Discussion

The conductivity is considered to be a log-normal field with a mean value of 1.5 MS/m and a coefficient of variance of 0.1. However, it was modified to be as a Gaussian field so that the KL expansion can be used since KL converges well with a Gaussian field. The modification will be discussed in detail in the full paper. The mean value of the interlaminar loss computed at 50 Hz using SSFEM and Monte Carlo method was 63 W and 65 W, respectively.

The computation time can be reduced significantly using SSFEM when compared to MC. The interlaminar loss was calculated using different stochastic approaches and an error of 3% was obtained from SSFEM when compared to MC at o=4 and M=4. The accuracy of the methods can be improved based on truncation of series. The error and sensitivity analysis on the computed loss will be presented in the full paper.

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